Ch. 6 Modern HW

Problem 3

Given a peak wavelength of 550 nm, determine the temperature of the radiating black body object.

$$\lambda_p T = 2.898 \text{ mm K}$$

$$T = \frac{2.898 \text{ mm K}}{550 \times 10^{-6} \text{ mm}} = 5269.1 \text{ K}$$

Ch6-6

Given that a temperature of a blackbody radiator changes so that the frequency is tripled, what happened to the temperature? And also energy density or power output?

$$\nu_{new} = 3\nu_0$$

$$\text{so} \quad T_{new} = 3T_0$$

$$\underline{\rho} \propto T^4$$

$$\text{so} \quad \frac{\rho_{new}}{\rho_0} = \left(\frac{T_{new}}{T_0}\right)^4 = 81$$

Problem Ch6-9

373 K

Given a 2mm^2 hole from a blackbody radiator with temp of 100C^o , how long does it take for 500J of energy to be emitted from the hole. Always assume emissivity is "1" if not given or stated.

$$\frac{\text{Energy}}{\text{time}} = \underline{P} = \sigma A \epsilon T^4 \qquad \sigma = 5.6705 * 10^{-8} \frac{W}{m^2 K}$$

$$\text{time} = \frac{500\ J}{\sigma (2 * 10^{-6} m^2)(373)^4}$$

DO IT

$$= 2.28 * 10^5\ s$$

6-13

Convert the frequency expression for energy density inside a blackbody cavity to one that depends on wavelength.

$$c = \nu\lambda \qquad \nu = \frac{c}{\lambda} \qquad d\nu = -\frac{c\,d\lambda}{\lambda^2}$$

$$U_\nu\, d\nu = \frac{8\pi h\nu^3/c^3}{\exp\left[\frac{h\nu}{KT}\right]-1}\, d\nu$$

now substitute

$$= \frac{\left(8\pi h\left(\frac{c}{\lambda}\right)^3/c^3\right)}{\exp\left[\frac{hc}{\lambda KT}\right]-1}\left(\frac{-c}{\lambda^2}\, d\lambda\right)$$

$$= \frac{8\pi h}{\lambda^5}\frac{(-c)\quad d\lambda}{\left[\exp\left[\frac{hc}{\lambda KT}\right]-1\right]}$$

Note that the minus sign is merely an indicator that when frequency increases, then wavelength decreases. So the sign is often ignored. Next task---**FIND THE PEAK**

Take the function, take the derivative and set = zero. Determine the wavelength that gives a peak (in terms of T and some number involving constants).

$$\frac{dU_\lambda}{d\lambda} = 8\pi hc\left(-5\lambda^{-6}\right)\left[\quad\right]^{-1}$$

$$+\frac{8\pi ch}{\lambda^5}(-1)\left(\left[\quad\right]^{-2}\right)\left(\frac{hc}{KT}*\frac{-1}{\lambda^2}\right)\left(\exp\left(\frac{hc}{KT\lambda}\right)\right)$$

Set equal to zero, get rid of common terms (divide out) things like $\frac{8\pi ch}{\lambda^5}$, $\left[\quad\right]^{-1}$

$$-5 + \frac{hc}{kT\lambda} \exp\left(\frac{hc}{kT\lambda}\right)\left[\quad \right]^{-1} = 0$$

I now call $u = \frac{hc}{kT\lambda}$

$$\underbrace{-5 + \frac{u \exp(u)}{\exp(u) - 1}}_{f(u) \,=\, 5} = 0$$

There is room for your algebra to look wildly different here. I could multiply out by denominator, call something different for "u".....none of this matters to the end result. I moved the "5" over to the right, and called the left f(u). I then take a spreadsheet of f(u) and u, and vary u until f(u) equals 5 .

Once you are close you should refine your search to a really fine grid and get u to about five or six digits of precision.

I get u=4.96511 (which is of course dimensionless).

$$4.96511 = \frac{hc}{kT\lambda_p}$$

the special λ that maximizes u_λ

Plug in constants and you will get

$$\lambda_p T = 2.898 \text{ mm K}$$

And now you have derived a fundamental law that you can actually check with the right spectroscopy equipment, and a hot glowing object.

ch 6 -17

Given that the surface temperature of the sun is approximately 5800K,

a) What is the peak frequency?

$$\nu_p = \frac{58.79\ GHz}{T}$$

$$= 3.41 \times 10^5\ GHz$$

b) What is the wavelength corresponding to the frequency found in part a --? NOTE this is not the peak wavelength!!!!!!!

$$\lambda_a = \frac{c}{\nu_p} = 8.798 \times 10^{-7} m$$

c) What is the peak wavelength (peak of the blackbody spectra for wavelength)

$$\lambda_p T = 2.898\ mm\ K$$

$$\lambda_p = 4.997 \times 10^{-7} m$$

Very diff

d) What is the frequency corresponding to part c

$$\nu_c = \frac{c}{\lambda_p} = 6.004 \times 10^{14}\ HZ$$

$$= 6.004 \times 10^5\ GHZ$$

e) The numbers differ due to the factor of $d\nu \propto \frac{d\lambda}{\lambda^2}$

f) Knowing the solar constant and orbital radius, determine the size of the sun.

$$I_{\substack{at \\ Earth}} \sim 1.39\ kWatt/m^2$$

$$r_{S-E} \simeq 1.50 \times 10^{11}\ m$$

$$I_{at}(r_{SE})$$

S ⊕ E

I_{Earth}

We have data to calculate the entire power received by a Dyson Sphere at the orbital radius of the Earth.

$$P = 1.39 \frac{kW}{m^2} * 4\pi r_{SE}^2$$

$$= \text{The output of sun} \quad \sigma, A T^4$$

$$I_{Earth} 4\pi r_{SE}^2 = \sigma 4\pi r_{sun}^2 T_{sun}^4$$

$$r_{sun} = 6.98 \times 10^8 \text{ m}$$ pretty close

Colbert Problem

What is the effective photonic mass (using E=mc^2) of all the radiant blackbody energy of the universe.   T~2.7K

$$r_{univ} \sim 13 \text{ billion LY} \qquad 1 \text{ LY} = 9.46 \times 10^{15} \text{ m}$$

$$I_{TOT} = \sigma T^4 = 3.01 \times 10^{-6} \frac{\text{Watts}}{m^2}$$

$$\frac{c}{4} (\text{Energy density}) = \frac{\text{Power}}{A}$$

$$\text{Energy density} = \frac{4}{c} \left(\sigma T_{univ}^4 \right)$$

$$= 4 \times 10^{-14} \text{ J}/m^3$$

$$\text{Energy} = 4 \times 10^{-14} \frac{J}{m^3} * \frac{4}{3} \pi r^3$$

$$= 3.1 \times 10^{65} \text{ J}$$

$$= M_{effective} c^2$$

$$M_{eff} = 3.46 \times 10^{48} \text{ Kg}$$

The mass of ordinary matter is estimated at about 10^(53 or 54) kg. So this photon mass is a lot.
Photons---"They're everywhere"   ---not so with ordinary matter.